SI Physics - Full Discipline Demo

Propagation of Uncertainty

Final Report - Answer Guide

Institution	Science Interactive University
Session	SI Physics - Full Discipline Demo
Course	SI Physics - Full Discipline Demo
Instructor	Sales SI Demo

Test Your Knowledge

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Match each term with the best description.

Terms:

- Absolute uncertainty
- Accuracy
- Error
- Precision
- Random error
- Relative uncertainty
- Systematic error
- True value

Description	
Known quantity that serves as a scientific reference	1
Size of the range of values which most likely includes the true value	2
Ratio of the absolute uncertainty of a measurement to the measured value	3
Difference between a measured value and the true value	4
Error resulting in measurements consistently higher or lower than the true value; also called determinate error	5
Error resulting in measurements with unpredictable high and low values; also called indeterminate error	6
Description of how closely a measured value is to the true value	7
Description of how closely two or more independent measurements agree with each other	8

Correct answers:

1 True value 2 Absolute uncertainty 3 Relative uncertainty 4 Error

5 Systematic error 6 Random error 7 Accuracy 8 Precision

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When uncertainties from multiple variables are combined, the accumulating error can be calculated using the following equations. Determine whether each equation for uncertainty applies to measurements derived through addition/subtraction, multiplication, division, or application of powers (exponents). Symbols in the figure represent measurements (*x*, *y*), uncertainty (∆*x*, ∆*y*), results/unknowns (*z*), and powers (*n*).

- Addition/Subtraction
- Division
- Function
- Multiplication
- Power

$$\Delta z = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 (\Delta x)^2 + \left(\frac{\partial f}{\partial y}\right)^2 (\Delta y)^2}$$

1

$$\Delta z = |xy| \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

2

$$\Delta z = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

3

$$\Delta z = |n| x^{n-1} \Delta x$$

4

$$\Delta z = \left|\frac{x}{y}\right| \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

5

Correct answers:

1 Function 2 Multiplication 3 Addition/Subtraction 4 Power

5 Division

Exploration

The size of the range of values in which the true value of the measurement probably lies is called the _____.

- absolute uncertainty ✔
- error
- accuracy
- All of the above

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Propagation of uncertainty is the _____.

- the exact value that exists in nature
- the size of the range of values in which the true value of the measurement probably lies
- the ratio of the absolute uncertainty of a measurement to the best estimate
- the mathematical technique to address uncertainty in results produced from errors in initial measurements ✔

For experiments with multiple measurements, the uncertainty of each measurement can simply be added to find the total uncertainty.

- True
- False ✔

When two measurements are added together, uncertainty is calculated with the equation _____.

- $\Delta z = (\Delta x)(\Delta y)$
- $\Delta z = \sqrt{(\Delta x)^2 + (\Delta y)^2}$ ✔
- $\Delta z = \sqrt{(\Delta x)^2 - (\Delta y)^2}$
- $\Delta z = |xy| \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$

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When two measurements are multiplied, uncertainty is calculated with the equation _____.

- $\Delta z = \sqrt{(\Delta x^2) - (\Delta y^2)}$
- $\Delta z = |xy| \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$ ✔
- $\Delta z = \left|\frac{x}{y}\right| \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$
- $\Delta z = |n|\, x^{n-1}\, \Delta x$

When two measurements are divided, uncertainty is calculated with the equation _____.

- $\Delta z = \sqrt{(\Delta x^2) - (\Delta y^2)}$
- $\Delta z = |xy| \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$
- $\Delta z = \left|\frac{x}{y}\right| \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$ ✔
- $\Delta z = |n|\, x^{n-1}\, \Delta x$

Exercise 1

Describe the similarities and differences between absolute uncertainty and relative uncertainty.

Both measure expected variability around a true value. Absolute uncertainty is an integer describing a range of values, while relative uncertainty is a ratio comparing absolute uncertainty to the estimated value of the measurement.

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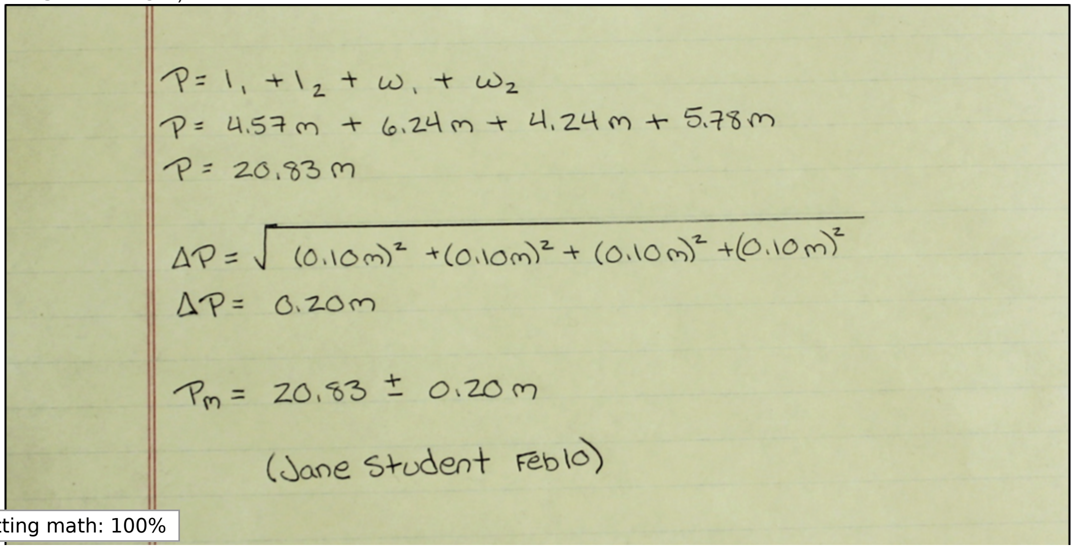
Data Table 1: Calculated Results
(SAMPLE ANSWER BELOW)

Problem	Result (Ex: 10.5 ± 0.2 L)
Problem 1: Find the perimeter of the fence, P_m.	$20.83 \pm 0.20\ m$
Problem 2: Find the perimeter of the shed, P_m.	$28 \pm 1\ ft$
Problem 3: Find the sum of masses, m_m.	$270 \pm 20\ kg$
Problem 4: Find the area of the basketball court, A_m.	$4704 \pm 94\ ft^2$
Problem 5: Find the speed of the bowling ball, s_m.	$7.62 \pm 0.35\ \frac{m}{s}$
Problem 6: Find the volume of the sugar cube, V_m.	$4.93 \pm 2.09\ cm^3$
Problem 7: Find the area of the triangle, A_m.	$72000 \pm 4400\ m^2$
Problem 8: Find the force of gravitational attraction, F_m.	$1.79\ x\ 10^{-13} \pm 1.14\ x\ 10^{-13}\ N$
Problem 9: Find the volume of an ellipsoid, V_m.	$4200 \pm 400\ cm^3$

Photo 1: Calculations for Problem 1
(SAMPLE ANSWER BELOW)


$P = l_1 + l_2 + w_1 + w_2$
$P = 4.57\,m + 6.24\,m + 4.24\,m + 5.78\,m$
$P = 20.83\,m$

$\Delta P = \sqrt{(0.10m)^2 + (0.10m)^2 + (0.10m)^2 + (0.10m)^2}$
$\Delta P = 0.20\,m$

$P_m = 20.83 \pm 0.20\,m$

(Jane Student Feb10)

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Photo 2: Calculations for Problem 2

(SAMPLE ANSWER BELOW)

$P = l_1 + l_2 + w_1 + w_2$

$P = 10.0\,ft + 5.0\,ft + 5.0\,ft + 8.0\,ft$

$P = 28.0\,ft$

$\Delta P = \sqrt{(0.5\,ft)^2 + (0.5\,ft)^2 + (0.5\,ft)^2 + (0.5\,ft)^2}$

$\Delta P = 1\,ft$

$P_m = 28 \pm 1\,ft$

(Jane Student Feb 10)

Photo 3: Calculations for Problem 3

(SAMPLE ANSWER BELOW)

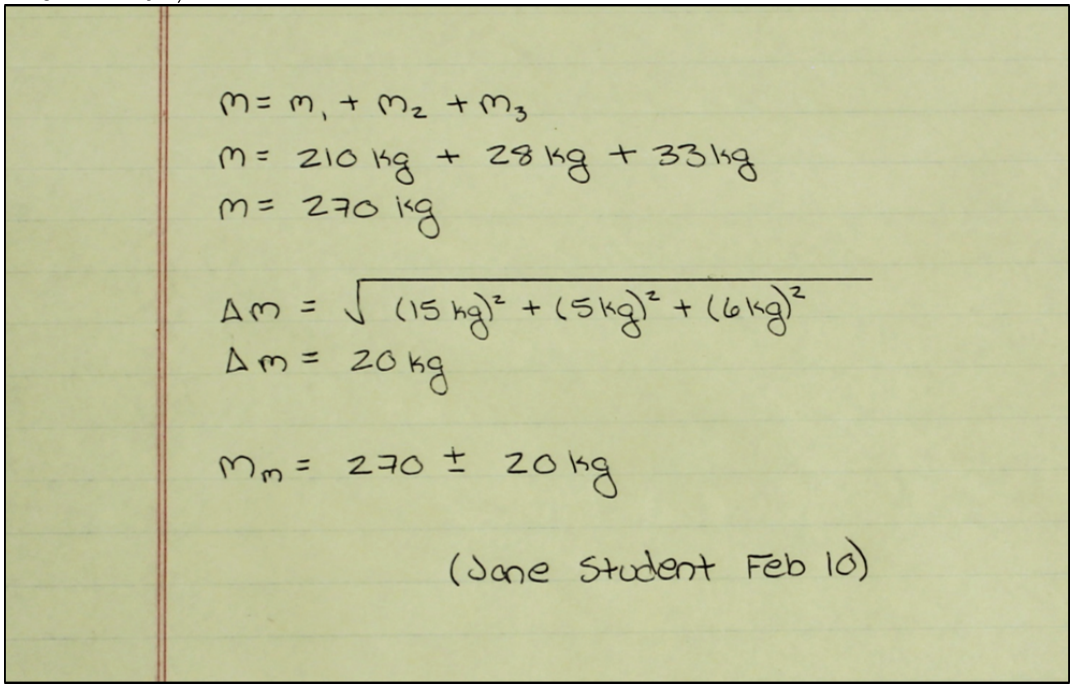

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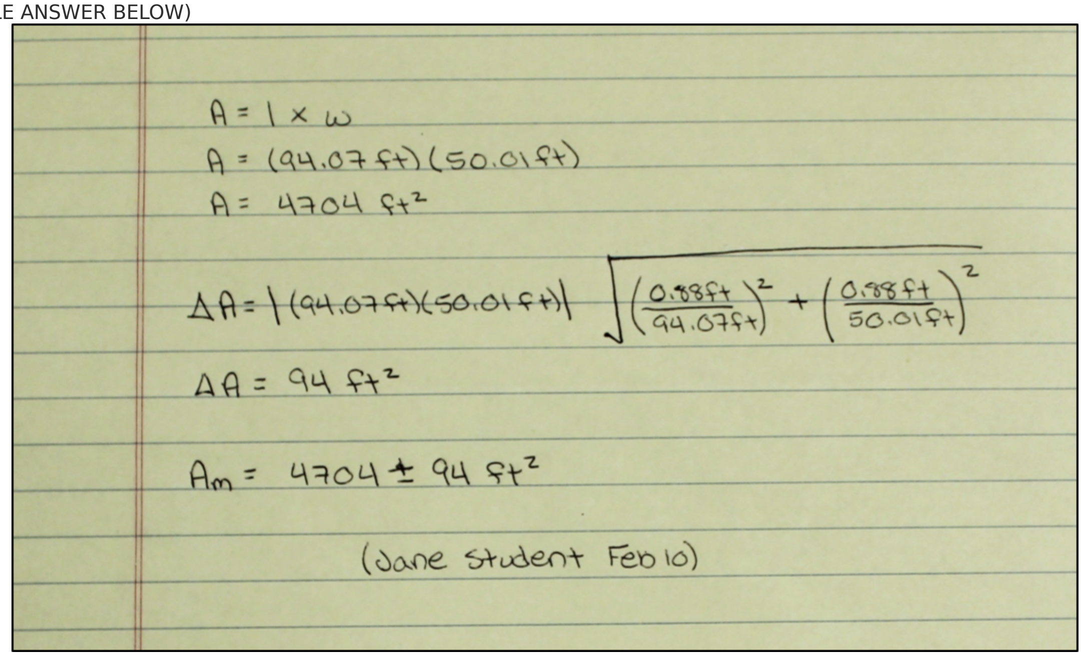
Photo 4: Calculations for Problem 4

(SAMPLE ANSWER BELOW)



$A = l \times w$

$A = (94.07\ ft)(50.01\ ft)$

$A = 4704\ ft^2$

$\Delta A = |(94.07\ ft)(50.01\ ft)| \sqrt{\left(\frac{0.88\ ft}{94.07\ ft}\right)^2 + \left(\frac{0.88\ ft}{50.01\ ft}\right)^2}$

$\Delta A = 94\ ft^2$

$A_m = 4704 \pm 94\ ft^2$

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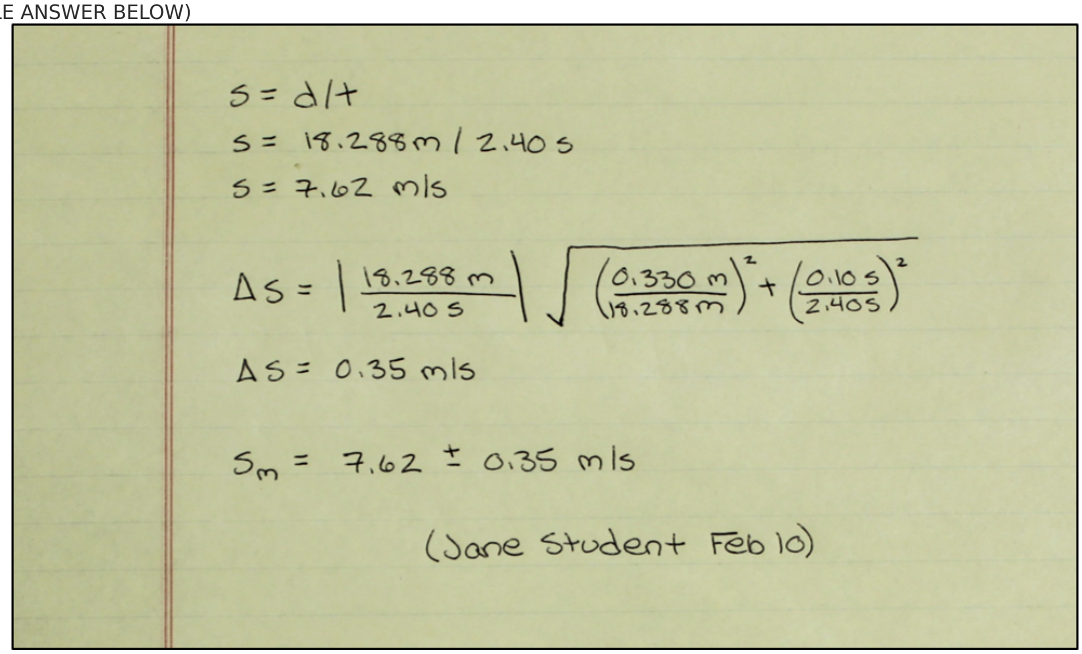
Photo 5: Calculations for Problem 5

(SAMPLE ANSWER BELOW)



$s = d/t$

$s = 18.288\,m / 2.40\,s$

$s = 7.62\ m/s$

$$\Delta s = \left|\frac{18.288\ m}{2.40\ s}\right| \sqrt{\left(\frac{0.330\ m}{18.288\ m}\right)^2 + \left(\frac{0.10\ s}{2.40\ s}\right)^2}$$

$\Delta s = 0.35\ m/s$

$s_m = 7.62 \pm 0.35\ m/s$

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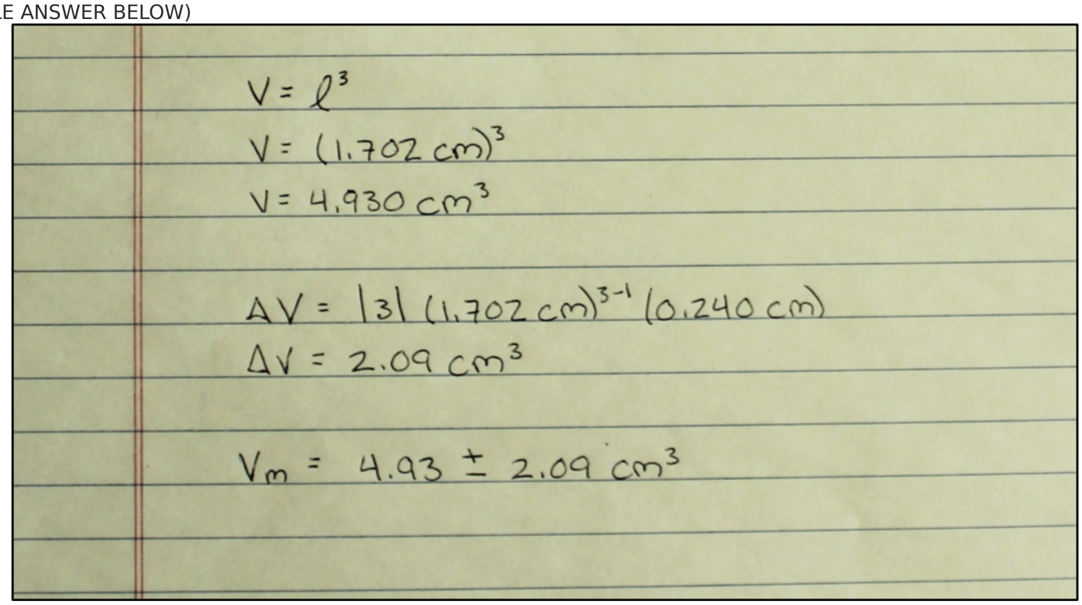
Photo 6: Calculations for Problem 6
(SAMPLE ANSWER BELOW)



$V = \ell^3$

$V = (1.702\ cm)^3$

$V = 4.930\ cm^3$

$\Delta V = |3|(1.702\ cm)^{3-1}(0.240\ cm)$

$\Delta V = 2.09\ cm^3$

$V_m = 4.93 \pm 2.09\ cm^3$

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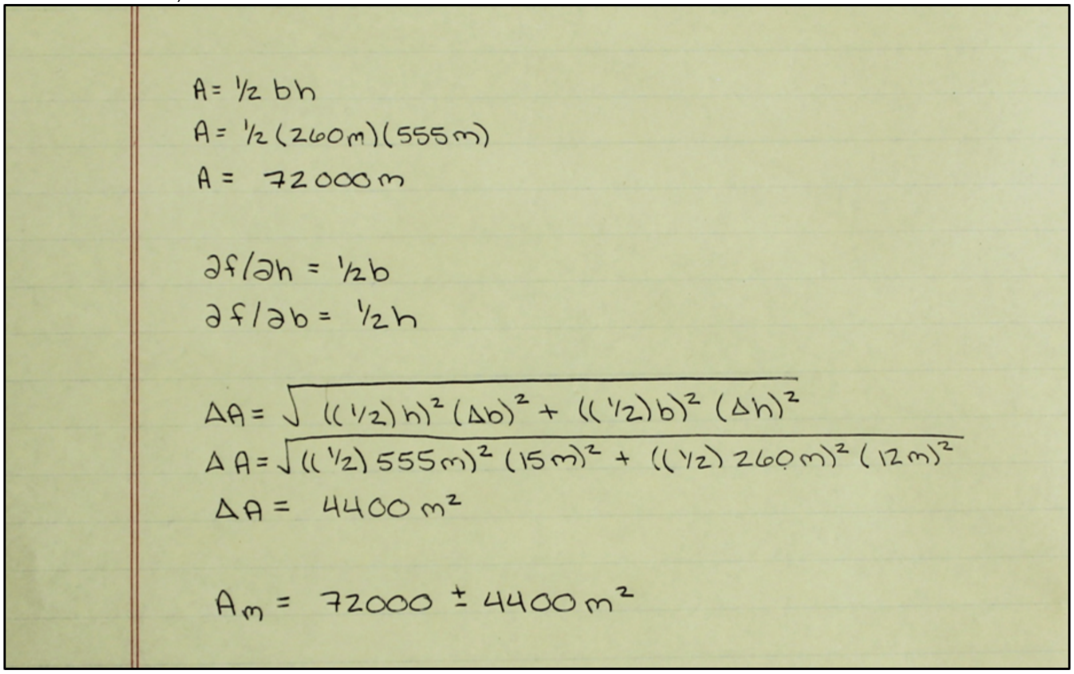
Photo 7: Calculations for Problem 7
(SAMPLE ANSWER BELOW)



$A = \frac{1}{2} bh$

$A = \frac{1}{2}(260m)(555m)$

$A = 72000m$

$\partial f / \partial h = \frac{1}{2} b$

$\partial f / \partial b = \frac{1}{2} h$

$\Delta A = \sqrt{((1/2)h)^2 (\Delta b)^2 + ((1/2)b)^2 (\Delta h)^2}$

$\Delta A = \sqrt{((1/2) 555m)^2 (15m)^2 + ((1/2) 260m)^2 (12m)^2}$

$\Delta A = 4400 m^2$

$A_m = 72000 \pm 4400 m^2$

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Photo 8: Calculations for Problem 8

(SAMPLE ANSWER BELOW)

$$F = Gm_1m_2/R^2$$

$$F = \frac{(6.67 \times 10^{-11}\ Nm^2/kg^2)(0.342\ kg)(0.816\ kg)}{(10.20\ m)^2}$$

$$F = 1.79 \times 10^{-13}\ N$$

$$\partial f/\partial m_1 = Gm_2/R^2$$

$$\partial f/\partial m_2 = Gm_1/R^2$$

$$\partial f\ \partial R = -2Gm_1m_2/R^3$$

$$\Delta F = G\sqrt{\left(\frac{m_2}{R^2}\right)^2(\Delta m_1)^2 + \left(\frac{m_1}{R^2}\right)^2(\Delta m_2)^2 + \left(\frac{-2m_1m_2}{R^3}\right)^2(\Delta R)^2}$$

$$\Delta F = G\sqrt{\left(\frac{0.816\ kg}{(10.20\ m)^2}\right)^2(0.100\ kg)^2 + \left(\frac{0.342\ kg}{(10.20\ m)^2}\right)^2(0.420\ kg)^2 + \ldots}$$

$$\ldots \left(\frac{(-2)(0.342\ kg)(0.816\ kg)}{(10.20\ m)^3}\right)^2(1.21\ m)^2$$

$$\Delta F = (6.67 \times 10^{-11}\ Nm^2/kg^2)(1.71 \times 10^{-3}\ kg^2/m^2)$$

$$\Delta F = 1.14 \times 10^{-13}\ N$$

$$F_m = 1.79 \times 10^{-13} \pm 1.14 \times 10^{-13}\ N$$

(Jane Student Feb 10)

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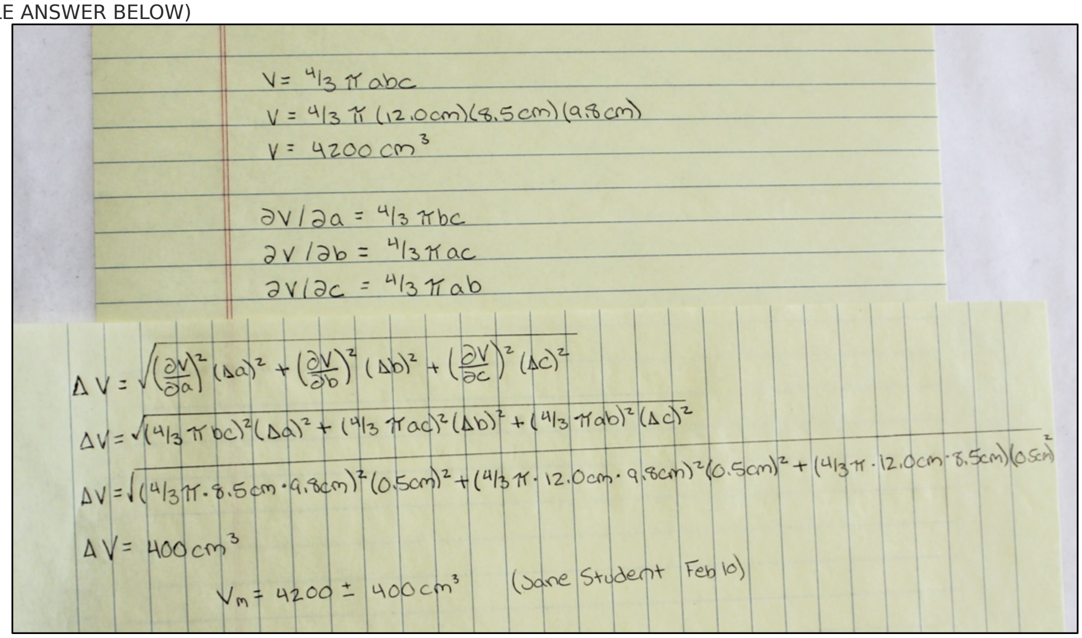
Photo 9: Calculations for Problem 9

(SAMPLE ANSWER BELOW)



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Competency Review

Absolute uncertainty is _____.

- exact value that exists in nature
- size of the range of values in which the true value of the measurement probably lies ✔
- ratio of the absolute uncertainty of a measurement to the best estimate
- mathematical technique to address errors in each initial measurement that produce uncertainty in the result

The “true value” is the exact value that exists in nature.

- True ✔
- False

The mathematical technique used to address uncertainty in results produced from errors in initial measurements is called _____.

- absolute uncertainty
- relative uncertainty
- propagation of uncertainty ✔
- All of the above

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When a final measurement is calculated using the equation $z = \frac{x}{y}$, the appropriate uncertainty equation shown in the figure is _____.

Equation 1	$\Delta z = \sqrt{(\Delta x)^2 + (\Delta y)^2}$
Equation 2	$\Delta z = \lvert xy \rvert \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$
Equation 3	$\Delta z = \left\lvert \frac{x}{y} \right\rvert \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$
Equation 4	$\Delta z = \lvert n \rvert x^{n-1} \Delta x$

- Equation 1
- Equation 2
- Equation 3 ✔
- Equation 4

If uncertainty is calculated with the equation $\Delta z = |n|\, x^{n-1}\, \Delta x$, the measurement was calculated using an equation with _____.

- addition
- multiplication
- a power ✔
- a function

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If two distances are measured as: 1.0 ± 0.1 m and 2.0 ± 0.1 m, the total distance (d_m) would equal _____.

- 1.0 ± 0.1 m
- 2.0 ± 0.1 m
- 3.0 ± 0.1 m ✔
- 3.0 ± 0.20 m

Extension Questions

Kirsten needs to create a solution composed of 150 mL of a chemical liquid. A variety of glassware is available. Table 1 describes the capacity and tolerance of each piece of glassware in Kirsten's laboratory. For this problem, let us assume that Kirsten's measurement techniques will be flawless and she will make no mistakes.

Table 1. Product information for transfer pipets and volumetric flasks, including capacity (total volume the glassware holds) and tolerance (uncertainty of measurement).

Glassware ID	Capacity (mL)	Tolerance (mL)
Pipet 01	1	± 0.006
Pipet 02	10	± 0.01
Flask A	25	± 0.03
Flask B	50	± 0.05
Flask C	100	± 0.08

1. Determine which combination of equipment would produce the best results:

a. Calculate the measurement with uncertainty for each of the following combinations. Use the addition rule

$$(\Delta z = |xy|\sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2 + \ldots}).$$

i. Combination 1 - Use "Flask C" 1 time; use "Pipet 02" 5 times:

ii. Combination 2 - Use "Flask B" 3 times:

iii. Combination 3 - Use "Flask C" 1 time; use "Flask B" 1 time:

iv. Combination 4 - Use "Pipet 01" 150 times:

b. Which combination of measurements would produce the best results? Use your calculations to explain your answer.

(SAMPLE ANSWER BELOW)

a) i: 150.00 ± 0.08 mL

ii: 150.00 ± 0.09 mL

iii: 150.00 ± 0.09 mL

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iv: 150.00 ± 0.07 mL

b) Combination 4: using “Pipet 01” 150 times produces the best results. This combination of measurements has the least uncertainty (± 0.07 mL), and therefore the measured value will be more accurate than any other combination of measurements.

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